

RECENT CONFIRMATION OF AN ASTRONOMICAL PREDICTION.

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(Read before the American Philosophical Society, October 3, 1873.)

In a communication to the Society on the 2d of May last, I gave certain harmonic indications of "a possible unknown planet, planetoid group, or other seat of solar and planetary perturbation" at about $\frac{7}{6}$ of the Earth's mean distance from the Sun.* I also suggested that Wolf's supposed sun-spot period of twenty-seven days "might be readily explained by the perturbations and transits of a planetoid or meteoric group, at a distance which would complete the terrestrial harmonic series." If there is any such source of perturbation, there should be not only maxima and minima of sun-spots at average intervals corresponding with the period of solar rotation, but there should also be subordinate maxima and minima at intervals of a half-rotation, inasmuch as the tidal influence would be exerted simultaneously at opposite extremities of the same solar diameter.

During my summer holidays I was enabled, through the courtesy of Prof. Joseph Winlock, Director of the Observatory of Harvard University, to examine observations of sun-spotted area, extending over a period of nearly five months. They furnished indications of such disturbances as I have described, but the period of observation was so short that I did not regard them as conclusive.

On my return to Philadelphia, I found in *Nature*, of July 17, an abstract of a communication to the Royal Society on June 19, by Messrs. De La Rue, Stewart and Loewy. Those eminent observers adduce evidences of a tendency in sun-spots "to change alternately from the north or positive to the south or negative hemisphere, and *vice versa*," and they attach special significance to the fact "that *the two outbreaks are at opposite ends of the same solar diameter*." These conclusions are based upon three sets of observations, taken in three different years, and extending over periods, respectively, of 145, 123, and 139 days. Their lowest approximate estimate of the mean interval between two maxima *of the same sign*, is 22.25 days; the highest, 28 days; "the most probable mean value, 25.2 days." The interval between two maxima of the same sign *and originating at the same axial extremity*, would of course be twice as great.

There seems, therefore, to be conclusive evidence of some disturbing force, revolving around the Sun in a period approximately equivalent to two solar rotations. The mean radius vector of such a disturbing force should be :

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* *Ante p. 238* ; see also *New York Tribune*. May 2, 1873.

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COMPARISON OF PLANETARY SERIES.

The closeness of the foregoing accordance may lend interest to a comparison of my own planeto-tactic series with those which have preceded it.

Jupiter, being about two and a-half times as great as the aggregate mass of all the other planets, may be regarded as forming with the Sun a binary star. The mutual action of the two controlling orbs of the system, is greatest at perihelion. A mean radius vector corresponding to the Schwabe sun-spot cycle, would be $(11.07)^{\frac{2}{3}} = 4.967$, which corresponds very closely with Jupiter's secular mean perihelion distance (4.9787, according to Stockwell, which would give a cycle of 11.11 years).

If the mean perihelion centre of gravity of the binary Star, Sun and Jupiter, be regarded as the extremity of a linear pendulum, of which the Sun's centre is the point of suspension, let us designate the distance of the centre of oscillation of the pendulum from the point of suspension by a .

$\pi \times a$	= major axis of solar orbit about <i>c. g.</i> of binary star.....	$= \beta$
$\pi^3 \times \beta$	= Mercury's mean perihelion distance.....	$= \gamma$
$\pi \times \gamma$	= Earth's “ “	$= \delta$
$\pi \times \delta$	= Asteroid at distance = $2 \times$ Mars.....	$= \varepsilon$
$\pi \times \varepsilon$	= Saturn's mean distance.....	$= \zeta$
$\pi \times \zeta$	= Neptune's “	$= \eta$
$\frac{2}{3} \eta$	= Uranus's mean aphelion distance.....	$= \theta$
$\frac{2}{6} \eta$	= Saturn's “ “	$= \iota$
$\frac{2}{5} \eta$	= ?	
$\frac{2}{12} \eta$	= Jupiter's mean perihelion distance.....	$= \kappa$
$\frac{1}{3} \kappa$	= Mars's mean aphehon distance.....	$= \lambda$
$\frac{1}{5} \kappa$	= Earth's mean distance.....	$= \mu$
$\frac{1}{7} \kappa$	= Venus's “	$= \nu$
$\frac{1}{9} \kappa$	= Mean of Mercury's and Venus's mean distances.....	$= \alpha$
$\frac{1}{11} \kappa$	= Mercury's mean aphelion distance.....	$= \pi$
$\frac{1}{13} \kappa$	= “ “ distance.....	$= \rho$
$\frac{1}{15} \kappa$	= “ “ perihelion distance.....	$= \varsigma$

It will be readily seen that all the terms of the foregoing centripetal

* α , assuming Jupiter's radial centre of oscillation as the fundamental unit; β , assuming Earth's mean distance as the unit.

series are in harmonic progression, and that the controlling influence of Jupiter over the progression is strikingly marked. A closer examination shows that the harmonic series which begins with Neptune's linear centre of oscillation is interrupted by the great masses of Jupiter and Saturn, and that the one which begins at Jupiter's mean perihelion is disturbed by the masses of Earth and Venus. The value of ν is very nearly an arithmetical mean between Venus's mean distance and her mean perihelion distance. The radius of Jupiter's mean linear centre of oscillation ($\frac{2}{3}$ of 5.2028 = 3.4685) is nearly equivalent to $\frac{2}{3}$ of Earth's mean radius vector, or to a radius vector (3.513) at the extremity of which, if the planetary masses were aggregated, the centre of gravity of the solar system would be at the source of the Sun's radiant undulation. From these three units we may derive the following harmonic series :

$\frac{1}{1} = 3.468$	$\frac{1}{1} = 3.500$	$\frac{1}{1} = 3.513$		
$\frac{1}{5} = .694$	$\frac{1}{5} = .700$	$\frac{1}{5} = .703$	Venus's mean perihelion	= .698
$\frac{1}{9} = .385$	$\frac{1}{9} = .389$	$\frac{1}{9} = .390$	Mercury's mean distance	= .387
$\frac{1}{13} = .267$	$\frac{1}{13} = .269$	$\frac{1}{13} = .270$	Radius of solar disturbance	= .267
$\frac{1}{17} = .204$	$\frac{1}{17} = .206$	$\frac{1}{17} = .207$	Vulcan ?	= .209

The value above given for Vulcan is the one which satisfies Kirkwood's estimated period for that supposed planet. If the actual values of Venus's mean perihelion and Mercury's mean distance be taken for two of the harmonic terms, and Jupiter's mean distance for the fundamental unit, we shall have the following harmonic series :

$1 \div 7.1343 =$	Venus's mean perihelion distance.
$1 \div 12.8604 =$	Mercury's " distance.
$1 \div 18.5865 = .2678 \times$	Earth's mean distance.
$1 \div 24.3126 = .2048 \times$	" "

There is, therefore, an apparent break in the harmonic progression, at a distance between .267 and .270 of the Earth's mean distance from the Sun, and within those limits we may reasonably look for the source of the peculiar sun-spot disturbances, which were pointed out by Messrs. De La Rue, Stewart and Loewy, in the communication above referred to.

The closeness of the approximations in my circular and harmonic series is shown in the following table. I take as the basis of my calculations, Bessel's valuation of Jupiter's mass ($1 \div 1047.879$), and Stockwell's estimate of Jupiter's mean eccentricity (.0431601). Sun's radius is taken as the unit. The error of the theoretical value is found by dividing the difference between the theoretical (T) and observed (O) values by the observed value.

CIRCULAR CENTRIFUGAL AND HARMONIC CENTRIPETAL SERIES.

	T.	O.	Error.
α	.680	.680	
β	2.136	2.132	+ .0020
γ	66.224	68.483	— .0330

	T.	O.	Error.
δ	208.048	207.583	+ .0023
ε	653.600	654.760	— .0018
ζ	2053.346	2049.514	+ .0019
η	6450.776	6453.731	— .0005
θ	4300.517	4306.465	— .0014
ι	2150.258	2148.609	+ .0008
κ	1075.129	1069.623	+ .0052
λ	358.376	353.263	+ .0145
μ	215.026	214.860	+ .0008
ν	153.590	155.415	— .0118
o	119.459	119.293	+ .0014
π	97.739	97.861	— .0012
ρ	82.702	83.172	— .0057
ς	71.675	68.483	+ .0466
Sum	18059.281	18053.947	+ .0003

The assumed "failure" of Bode's series, in the case of Neptune, disappears if we make the series symmetrical, as I have already suggested, by introducing two equal differences at the outer, as well as at the inner limit of the planetary belt. In the following table, Earth's mean distance is taken as the unit.

BODE'S SERIES, MODIFIED.

	T.	O.	Error.
$\S m^*$.4	.387	+ .0333
$\P p$.7	.698	+ .0029
$\oplus m$	1.0	1.000	.0000
$\S a$	1.6	1.644	— .0269
$2\text{I} m$	5.2	5.203	— .0005
$\text{I} a$	10.0	10.000	.0000
$\S m$	19.6	19.183	+ .0217
Ψp	29.2	29.735	— .0189
Sum	67.7	67.850	— .0022

The following table exhibits Peirce's phyllotactic series of sidereal periods† in the form in which the original investigation was made by Dr. Thomas Hill, in 1849, at Professor Peirce's request. For convenience of comparison with other series, I give not only the error (a) of time, but also the corresponding errors (β) of mean distance, (γ) of mean or perihelion distance.

* m , mean; a , mean aphelion; p , mean perihelion.

† Proc. Amer. Assoc., v, 2. Cambridge meeting.

PEIRCE'S PHYLLOTACTIC PLANETARY SERIES.

	T.	O. [60127]	Error α	Error β	Error γ
Ψ					
$\odot \frac{1}{2} \Psi$	30063	30687	— .0203	— .0136	m — .0136
$\frac{1}{2} \odot$	10229	10759	— .0493	— .0331	p + .0152
$\mathcal{U} \frac{2}{3} \frac{1}{2}$	4304	4333	— .0067	— .0045	m — .0045
Ast. $\frac{3}{8} \mathcal{U}$					
$\odot \frac{5}{13}$ Ast.	625	687	— .0902	— .0611	p + .0180
$\oplus \frac{1}{2} \odot$	343	365.26	— .0609	— .0410	p — .0071
$\odot \frac{8}{13} \oplus$	224.7	224.7	.0000	.0000	m .0000
$\otimes \frac{2}{3} \odot$	89.88	87.97	+ .0217	+ .0144	m + .0144
Sum	45878.58	47143.93	— .0269	— .0180	— .0033

Chauncey Wright, in the *Mathematical Monthly*, vol. 1, p. 244, referred the phyllotactic law to nodes of extreme and mean ratio, which he called "the *distributive ratio*." The same law which distributes leaves most evenly around the stem, would distribute planetary perturbations most evenly around the Sun.

Kirkwood (*Proc. Amer. Phil. Soc.*, v. 12, p. 163) gave a harmonic series which differed from Peirce's phyllotactic series by the omission of the asteroidal term, and by the substitution of $\frac{1}{6} \mathcal{U}$, $\frac{1}{12} \mathcal{U}$, and $\frac{2}{3} \oplus$, for $\frac{5}{13}$ Ast., $\frac{1}{2} \odot$, and $\frac{8}{13} \oplus$, in the expressions for the periodic times of Mars, Earth, and Venus, respectively. His approximations were closer than Peirce's for Mars and Earth, but not so close for Venus. The omission of any terms which depend directly upon the asteroidal belt and upon Mars, renders his series less symmetrical than Peirce's.

In the following table, the errors of the closest planetary approximations in each series are given for the purpose of comparison :

ERRORS OF THEORETICAL PLANETARY POSITIONS.

	Bode.	Peirce.	Kirkwood.	Chase.
\otimes	+ .0333	+ .0144	+ .0144	— .0057
\odot	+ .0029	.0000	+ .0001	— .0118
\oplus	.0000	— .0071	— .0079	+ .0008
\odot	— .0269	+ .0180	+ .0337	+ .0145
\mathcal{U}	— .0005	— .0045	— .0045	+ .0052
$\frac{1}{2}$.0000	+ .0152	+ .0152	+ .0008
\odot	+ .0217	— .0136	— .0136	— .0014
Ψ	— .0180	.0000	.0000	— .0005
Sum	— .0022	— .0018	— .0015	+ .0001

All of these approximations are so close as to preclude the idea of merely accidental coincidence, and to encourage an attempt to find some causal nexus through which they may all be referred to the law of gravitation.

Peirce's series has the special merit of being the first for which any reason was given. It represents the *mean* planetary distances more nearly than either of the other series, and if we accept the nebular

hypothesis, we may perhaps regard it as indicative of the initial tendency to planetary aggregation, inasmuch as we may reasonably suppose that the phyllotactic, or "distributive," ratio was most operative when the nebulous diffusion was greatest.

Bode's series, like Kepler's laws, seems to have been at first merely empirical. But if we conceive a rotating nebulous mass, with a slight equatorial nucleus of condensation, the line of particles between the nucleus and the centre will be influenced by tendencies to two different kinds of motion: first, as portions of a rotating mass, with velocities varying as the distance; second, as revolving particles, with velocities varying inversely as the square root of the distance. The first of these tendencies, combined with the moment of inertia, would urge such particles as were free to move, towards the linear centre of oscillation. If Mercury's mean distance be taken as the point of suspension, Uranus is situated approximately at Neptune's linear centre of oscillation, and each of the planets between Uranus and Venus, is at the linear centre of oscillation between the next superior and the next inferior planet.

My own series is based on the hypothesis that undulations excited in the elastic æther, like those of air, strings, and other elastic bodies, tend to produce harmonic as well as equivalent undulations. The orbital motions of planets, may be regarded as tangential undulations which have been deflected into circles by the continual influence of gravity, and the equivalent radial undulations seem to have determined the π series of groupings in planetary pairs, while the harmonic undulations have been most operative within the planetary belt, where the planetary and æthereal vibrations are in constant mutual inter-action. It will be seen that the mean error of Bode's eight terms (.0022), is more than seven times as great as that of my sixteen terms (.0003), and nearly twenty-seven times as great as that of the eight terms in my series which are directly comparable to his (.0000822). I can think of no mode of gravitating action which will account for such close accordance, as well as for the modifications of the harmonic series by planetary mass and the confirmation of my harmonic prediction, except the one I have already mentioned, the influence of equivalent and harmonic vibrations in an indefinitely elastic æther.

All of the terms in each series, except the Bodeian term for Neptune, represent *actual planetary positions*, or positions within the secular variations of planetary eccentricity. Stockwell's discussion of the secular variations of the orbital elements of the eight principal planets, has shown that the most important of the correlated apsides are the same as are represented in my harmonic series, for he has pointed out the following curious relations:

"I. The mean motion of Jupiter's perihelion is exactly equal to the mean motion of the perihelion of Uranus, and the mean longitudes of those perihelia differ by exactly 180° . II. The mean motion of Jupiter's node on the invariable plane is exactly equal to that of Saturn, and the mean longitudes of these nodes differ by exactly 180° ."

Jupiter's perihelion is therefore properly comparable with the aphelion of Uranus. A like comparability with Saturn's aphelion is not immediately evident, but "the orbit of Saturn is affected only by the *difference* of the perturbations by Jupiter and Uranus;" Saturn and Jupiter always act on the inclination of the orbit of Uranus in opposite directions; "the eccentricity of the orbit of Saturn always increases, while that of Jupiter diminishes, and *vice versa*;" therefore while either apsis of one of the planets is approaching the Sun, the corresponding apsis of the other planet is receding and the opposite apsis approaching. Then, if we look only to secular mean positions, Jupiter and Uranus are simultaneously in opposite apsides; and opposite apsides of Jupiter and Saturn are simultaneously approaching to, or receding from, the Sun.

The fundamental centre of oscillation which forms the unit of the π series, is determined by Jupiter's mean perihelion, with which, as we have just seen, the mean aphelia of Saturn and Uranus are correlated, and the same harmonic relations which subsist between these three important planets, fix, with close approximation, cardinal positions of the other planets.

If planetary positions and times have been determined by mutual gravitating action, it seems probable that in planetary masses and in the normal undulations of the æther itself, simple relations to the same action may be traceable. Stockwell says (p. xvii), "a comparison of the values * * * has suggested the inquiry whether there may not be some unknown physical relation between the masses and mean distances of the different planets." I believe that Proctor, and probably other astronomers, have also suggested such a relation, but I am not aware that one was ever pointed out until I called the attention of the Society (Proc. Amer. Phil. Soc., xiii, 141) to the equality between the mean moments of inertia of the two principal planets (Jupiter, $5.2028^2 \times 9543 = 258318$; Saturn, $9.5389^2 \times 2856 = 259851$).

Of the many correlations of light and gravity which I have hitherto indicated, the two following seem especially interesting in this connection. 1. The synchronism between the passage of a luminous undulation from the Sun to the centre of oscillation of the outermost planet, and the rotation of the Sun on its axis; 2. The equality of the limiting velocity of rotation to the velocity of light. In my original announcement of this equality, I introduced, as one of the elements of my calculation, the Sun's centre of spherical gyration. To this the very reasonable objection has been urged, that we know nothing of the internal density of the Sun. I therefore submit the following considerations, which are entirely independent of solar density.

In particles moving freely about an attracting centre, the mean $(4r \div t)$ velocity of oscillation through the centre of gravity, is to the velocity $(2\pi r \div t)$ of synchronous circular revolution about the centre of gravity, as 2, is to π . The velocity acquired by falling, from an infinite distance, to the extremity of any radius, is $\sqrt{2gr}$. If we suppose that the velocities of free oscillation and revolution are each retarded in the same pro-

portion at the surface of the rotating Sun, and designate the mean velocity of retarded oscillation by v_1 ; the velocity of infinite fall by v_2 ; under any assumed expansion or contraction of the Sun's mass,

$v_1 \propto \frac{1}{r}$,* while $v_2 \propto \sqrt{\frac{1}{r}}$. If we wish to find the limiting velocity v^0 for the value of r , ($r^0 = r \div x^2$), which renders $v_1^0 = v_2^0$, we have the equation

$$v^0 = v_1^0 = v_2^0 = v_1 x^2 = v_2 x$$

$$\therefore x = v_2 \div v_1$$

Taking Norton's estimate of solar radius (425,061.5) the value of v_2 is 375.5 miles per second, and various estimates of the time of solar rotation give the following values of v_1 and v_0 .

	v_1	v_0
Spörer.....	.799	176,440
Carrington.....	.788	178,920
Lelambre.....	.787	179,210
Faye.....	.785	179,660
Petersen.....	.781	180,470
De La Rue, Stewart and Loewy.....	.781	180,560
Herschel, (Bianchi and Langier).....	.777	181,460
Harmonic prediction ($\frac{7}{25} \oplus$).....	.771	182,800

The velocity of light, as deduced from Norton's value of solar radius, is 183,450 miles per second, which is approximately identical with the limiting velocity, v_0 .

TRANSCRIPT OF A CURIOUS MANUSCRIPT WORK IN CYPHER, SUPPOSED TO BE ASTROLOGICAL.

BY PLINY EARLE CHASE.

(*Read before the American Philosophical Society, October 3d, 1873.*)

The work, of which I have prepared the accompanying transcript, was bought in Amsterdam, about seventy years ago. It consists of forty-four manuscript pages, on twenty-four triangular leaves of parchment, measuring nine inches on a side, substantially bound in a hog-skin cover. It has been kept in a plush-lined tin case, so that it is in a state of excellent preservation, and appears as if newly written. On the lid of the case is a figure of a dragon, together with the following inscription, greatly defaced, but still distinct enough to be perfectly legible.

"EX DONO SAPIENTISSIMI COMITIS ST GERMAIN QUI ORBEM TERRARUM PERCUCURRIT."

The cypher consists of twenty-six arbitrary characters. In preparing to transcribe it, I counted the number of times each character was used, substituting a for the one that occurred most frequently, b for the next in frequency, and so on. The words are often run together, but there are numerous breaks, which I have indicated, some of which appear to mark divisions between words, while others may be arbitrary, or intended as blinds.

* In consequence of the law of equal areas.